ORDINAL LOGISTIC HLM

Timothy Olsen

HLM II – Dr. Gagne's Class

ABSTRACT

Ordinal Logistic Hierarchical Linear Modeling (HLM) is a method that has seen limited use due to its complex nature, and lack of software to properly model it. Recently, modeling capabilities have been added to the HLM software (Raudenbush, Bryk, & Congdon, 2004) which allow easy modeling of ordinal dependent variables in a logistic fashion. I describe the method, convey an interpretation of coefficients from a sample study, and discuss other research which has employed the method.

Keywords: ordinal logistic HLM, research methods, hierarchical linear modeling

I. DESCRIPTION OF METHOD

In much of research our outcome variables are considered to be continuous with the range of observed sample. Examples of such outcomes are miles per gallon, weight, height, test scores, etc. Sometimes the outcome is a mutually exclusive or binary (0 or 1) measure, such as cheating or not cheating, graduating or not graduating, approving of Pinochet or not approving of Pinochet. This analysis is easily done with Logistic regression.

Many interesting dependent variables do not involve an either or proposition, but rather some meaningful ordinal response. A primary characteristic of ordinal data is that a number assigned to successive categories can represent varying orders of magnitude, or a greater than, less than quality (Stevens 1946). These scales can be found in many places in education and social science research. Examples in education might include grades (0 = F, 1 = D, 2 = C, 3 = B, 4 = A), or levels of quality work (0 = un-satisfactory, 1 = satisfactory, 2 = excellent). Examples in other fields might include injury sustained in car accidents (0 = mild, 1 = moderate, 2 = severe) or alcohol problems of parents (0 = none, 1 = some, 2 = several).

The level of difficulty in analysis of comparing two levels is much more complex for ordinal than binary dependent variables. In a binary scale, the probability of graduating on time is the inverse of the probability of not graduating on time. When this is extended to an ordinal scale of graduating on time, graduating late, and not graduating at all the same inverse proportionality rule does not apply.

Because this analysis is more complex, there is little published research which use it. A Google search for “ordinal logistic HLM” yields three results, which are publications from the researchers at UCLA who did a study on test assessments which favor English learners (cite). I will discuss this paper in Section 4.

The methodologist who has written the most in this area is Hedeker, however his writings are at the multilevel statistic level, and not specific to the HLM software. Hedeker has a seminal paper in this area (Hedeker & Gibbons, 1994), and a more recent book chapter which explains the concept of thresholds quite well (Hedeker, 2007). His homepage has a freely-available copy of this book chapter, and other resources (“Multilevel Analysis - Don Hedeker,” 2009).

II. EXAMPLE

I will illustrate this using a serious parenting and social issue, child punishment. The design of this hypothetical study consists of a survey sent to parents in each state of the United States. The survey asked each parent what their level of acceptability of physical punishment for children was (3 = high, 2 = medium, 1 = low). It also asked them gender (0 = female, 1 = male), and their educational level (0 = no BS degree, 1 = BS degree or higher). In addition, state government statistics of number of deaths resulting from child punishment were recorded in the level-2 state data.

We would like to see what effect gender and educational level, and have on the personal level of acceptability of physical punishment for children. We also would like to see what affect the state level has.

Putting this into HLM for Windows (Raudenbush et al., 2004), gives us these Level 1 and Level 2 equations:

Level 1 Equation

Prob[R = 1|B] = P'(1) = P(1)

Prob[R <= 2|B] = P'(2) = P(1) + P(2)

Prob[R <= 3|B] = 1.0

where

P(1) = Prob[ChildPunishmentAcceptability(1) = 1|B]

P(2) = Prob[ChildPunishmentAcceptability (2) = 1|B]

log[P'(1)/(1 - P'(1))] = B0 + B1*(GENDER) +B2*(EducationLevel)

log[P'(2)/(1 - P'(2))] = B0 + B1*(GENDER) +B2*(EducationLevel) + d(2)

Note: Differing from Logistic regression, cumulative probabilities are at play here, this explains the three different probability functions. The first probability function is the probability of being in category 1. The second probability function is the probability of being in category 2 or 1. The third probability function is the probability in being in any category, and thus equal to 1. The 'd(2)' at the end of the last equation signifies the difference necessary to adjust from the first probability function, to the second.

Level 2 Equations

$b_{0j} = \gamma_{00} + \gamma_{01}$·Num_deaths$_j$ + + u_{0j}

$b_{1j} = \gamma_{10}$

$b_{2j} = \gamma_{20}$

d(2)

Table 1. Acceptability of Physical Punishment of Children		
		Coefficients (log-odds)
Intercept	γ_{00}	-1.78
Female	$\gamma 10$	-0.32
Education	$\gamma 20$	-0.24
Num_Deaths	γ_{01}	0.3
Threshold d(2)		0.86

Interpreting coefficients in Ordinal Logistic HLM, is the same as interpreting coefficients in logistic HLM. However, in Ordinal Logistic HLM there is one new thing the Threshold level. There will always be n-2 threshold for every ordinal category. Therefore we only one threshold in this example.

Negative Coefficients can be interpreted decreasing likelihood of being accepting of physical punishment of children. Positive coefficients can be interpreted as increasing likelihood of being accepting. We can interpret the coefficient of $e^{\gamma 00}$ by transforming it from log-odds to probability form: $p = e^{\gamma 00} / (1 + e^{\gamma 00})$. Doing so yields .14, which we can interpret as %14 of all people sampled have a very high acceptability of child punishment.

Threshold levels give you the probability of n-1 categories. In our case this is high (=1) and medium (=2) categories, the bottom two categories. By adding this coefficient to the transformation of the log-odds scale we can receive the probability the bottom two

levels: $p = e^{\gamma 00+\delta(2)} / (1 + e^{\gamma 00+\delta(2)}) = p = e^{-1.78+.86} / (1 + e^{-1.78+.86}) = 28\%$. By derivation the probability that respondents had a low level of acceptability is 72%.

The other coefficients can be easily interpreted as probabilities in like manner. As a rule of thumb, negative coefficients are indicative of a decreasing likelihood of viewing physical child punishment as acceptable. Thus, as the number of deaths (per million) due to child maltreatment goes up by one, the likelihood of the state's inhabitants to have higher acceptability levels rises.

III. METHODOLOGICAL ISSUES

Ordinal Logistic HLM has similar sample size and other issues which effect logistic HLM, and HLM. The mathematics behind ordinal logistic HLM are quite a bit more complex than logistic HLM.

At this point, I am unaware of other issues which are specific to this method.

IV. EXAMPLES FROM THE LITERATURE

There are not many examples from the literature which use ordinal logistic HLM methods. However, there are a few great examples that are simple to understand and well written, I will discuss these papers in the "Exemplars" section below, other research which uses multilevel methods and ordinal dependent variables I will discuss in the "Others" section below.

EXEMPLARS

Two researchers from Universities in Spain have written two papers which are well written and good exemplars of the method. Their first work (Gracia & Herrero, 2008) is a

multilevel study of 10,812 people nested within 208 cities in 14 countries of the European Union. The dependent or “outcome variable” which they measure is three category ordinal measure of acceptability of physical punishment for children. They collect several individual level measures such as gender, education level and age. They also use country data about the number of deaths from child maltreatment (per million people), and a binary measure of if physical punishment was banned in that country by 1999. Their results section is well written and is very helpful to the amateur methodologist. As their ordinal scale consists of three items they have report one threshold level. Their use of the method is cleanly done, and simple to understand.

The other work (Gracia & Herrero, 2006) is similar, but deals with acceptability of domestic violence against women (DVAW). Their data comes from 13,475 people nested in 212 cities within 15 countries of the European Union. The dependent variable consists of a four level ordinal measure of acceptability of domestic violence against women (1 = always unacceptable, 4 = always acceptable). Their notable results include significant effects of victim blaming, and perceived lack of DVAW as a problem both of which lead to an increased likelihood in acceptability of DVAW.

Another great exemplar includes the work of the UCLA researchers who are interested the factors involved in the achievement gap between English Learners and non-English Learners in elementary schools (Boscardin, Aguirre-Munoz, Chinen, Leon, & Shin, 2004).

Their sample consists of 1,038 6th grade students taught by 27 different teachers who are a part of 7 elementary schools in Los Angeles, California. Each student was administed a Language Arts Performance Assignment in which they were asked to

choose a literary work which contains a heroic character and describe the character in writing, citing the text. These assignments were then rated by their teachers who assigned four point grade to their work (1 = sub-standard, 4 = best). This was used as the dependent or outcome variable. At the teacher level data was collected on subject matter expertise and classroom time spent on literary analysis and writing. This paper is unique in that it used raters to determine the ordinal values, rather than perceptual measures.

OTHERS

Other papers do not include threshold values and are less clear about their use of HLM. One example is a public health study from Taiwan. This study is motivated by the fact that although women live longer than men on average, they report poorer health. (Chen, Chang, & Yang, 2008) investigated how gender roles derived from personal and household factors may affect perceived health measures. Their sample consists of 50,354 respondents in 13,000 households in 867 villages. The ordinal dependent variable consists of a perceived health measure (1 = good, 4 = poor and not capable of self-care). This study is interesting, however as they do not list threshold levels it is difficult to see the probabilities groups. Another similar study breaks a binary measure of prior consumption of alcohol and no prior consumption of alcohol (Med, 2004).

REFERENCES

Boscardin, C. K., Aguirre-Munoz, Z., Chinen, M., Leon, S., & Shin, H. S. (2004). Consequences and Validity of Performance Assessment for English Learners: Assessing Opportunity to Learn (OTL) in Grade 6 Language Arts. CSE Report 635. *Center for Research on Evaluation Standards and Student Testing CRESST*, 78.

Chen, D. R., Chang, L. Y., & Yang, M. L. (2008). Gender-specific responses to social determinants associated with self-perceived health in Taiwan: A multilevel approach. *Social Science & Medicine*.

Gracia, E., & Herrero, J. (2006). Acceptability of domestic violence against women in the European Union: a multilevel analysis. *Journal of Epidemiology & Community Health*, *60*(2), 123.

Gracia, E., & Herrero, J. (2008). Is It Considered Violence? The Acceptability of Physical Punishment of Children in Europe. *Journal of Marriage and Family*, *70*(1), 210-217.

Hedeker, D. (2007). Multilevel models for ordinal and nominal variables. *Handbook of Multilevel Analysis*.

Hedeker, D., & Gibbons, R. D. (1994). A random-effects ordinal regression model for multilevel analysis. *Biometrics*, *50*(4), 933-44.

Med, K. J. P. (2004). A Study of the Relationship between Parental Alcohol Problems and Alcohol Use among Adolescent Females in Republic of Korea. *Korean J Prev Med*, *37*(3), 232-237.

Multilevel Analysis - Don Hedeker. (2009). . Retrieved February 21, 2009, from http://tigger.uic.edu/%7Ehedeker/ml.html.

Raudenbush, S., Bryk, A., & Congdon, R. (2004). *HLM for Windows*. Version 6.06.